Theorem Let a and b be positive integers such that $a > b$ and let $r_K = 0$ in Euclidean algorithm, then r_{K-1} is the g.c.d of a and b.

Proof:→ We know that

$$r_{K-2} = r_{K-1} q_K \Rightarrow r_{K-1} \mid r_{K-2}$$

Again we have

$$r_{K-3} = r_{K-2} q_{K-1} + r_{K-1}$$

$$= r_{K-1} q_K q_{K-1} + r_{K-1}$$

$$= r_{K-1} [q_K q_{K-1} + 1]$$

which implies that

$$r_{K-1} \mid r_{K-3}$$

Continuing this process finally we get

$$r_{K-1} \mid a \text{ and } r_{K-1} \mid b$$

Now let c divides a and b. Since $a = bq_1 + r_1$ which then c divides b and r_1. Also $b = r_1 q_2 + r_2$ which implies c divides r_1 and r_2. Continuing the process, we get c divides r_{K-1}

Hence g.c.d $(a, b) = r_{K-1}$

Numerical Find the greatest common divisor of 525 and 231.

Solution We have

$$525 = 2 \times 231 + 63$$

$$231 = 3 \times 63 + 42$$

$$63 = 1 \times 42 + 21$$

~~42 = 2×21~~ remainder = 21

Hence $(525, 231) = 21$

231) 525 (2
462
63) 231 (3
189
42) 63 (1
42
21) 42 (2
42
××

Numerical Find the g.c.d of 396 and 671

Solution We can write

$671 = 2 \times 396 - 121$ $= (671, 396)$

$396 = 121 \times 3 + 33$ $= (396, 121)$

$121 = 4 \times 33 - 11$ $= (121, 33)$

$33 = 3 \times 11 + 0$ $= (33, 11)$

Hence $(396, 671) = 11$

396) 671 (2
792
121) 396 (3
363
33) 121 (4

Example Find the greatest Common divisor of 42823 and 6409

Solution Here we apply the euclidean algorithm. We divide c into b where b = 42823 and c = 6409 is divided in to 42823, we get 6.68 therefore quotient is 6. To get the remainder, we multiply 6 by 6409 to get 38454 and we subtract this from 42823 to get the remainder 4369. Thus $q_1 = 6$ and $r_1 = 4369$. Continuing the same process, if we divide 4369 in to 6409, we get a quotient $q_2 = 1$ and remainder $r_2 = 2040$. Further dividing 2040 into 4369 gives $q_3 = 2$ and $r_3 = 289$. Again dividing 289 in to 2040 gives $q_4 = 7$ and $r_4 = 17$, Since 17 is a divisor of 289 the solution is that g.c.d is 17.

$42823 = 6(6409) + 4369$

$6409 = 1(4369) + 2040$

$4369 = 2 \times 2040 + 289$

$2040 = 7(289) + 17$

$289 = 17(17)$

$\therefore$ g.c.d = 17

New Find the L.c.m of 136, 221, and 391

Solution We have

$$[136, 221, 391] = [[136, 221], 391]$$

$$= \left[\frac{136 \times 221}{17}, 391\right]$$

$$= [1768, 391]$$

$$= \frac{1768 \times 391}{17} = 40664$$

Example Find the L.c.m of 8, 12, 20 and 25

Solution using $[a,b] = \frac{ab}{(a,b)}$

$$[8,12] = \frac{8 \times 12}{(8,12)}$$

$$= \frac{96}{4} = 24$$

$$[24,15] = \frac{24 \times 15}{(24,15)} = \frac{24 \times 15}{3} = 120$$

$$[120, 20] = \frac{120 \times 20}{(120,20)} = \frac{120 \times 20}{20} = 120$$

$$[120, 25] = \frac{120 \times 25}{(120,25)} = \frac{120 \times 25}{5} = 600$$

Hence L.c.m of 8, 12, 15, 20, and 25 is given by

~~8,14~~ $[8, 12, 15, 20, 25] = 600$

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